Book of abstracts of the 6th International Conference on Advanced Computational Methods in Engineering, ACOMEN 2014
23–28 June 2014.

Understanding Population Balances Involving Aggregation and Breakage Through Homotopy Approaches

Zehra Pinar[1], Abhishek Dutta[2], Aniruddha Majumder[3], Denis Constales[4] and Turgut Öziş[5]

[1] *Department of Mathematics, Faculty of Arts and Science, Namık Kemal University, 59030 Merkez-Tekirdağ, Turkey*

[2] *Departement Metaalkunde en Toegepaste Materiaalkunde (MTM), KU Leuven, Kasteelpark Arenberg 44, B-3001 Heverlee-Leuven, Belgium*

[3] *Department of Chemical Engineering, Loughborough University, Loughborough LE11 3TU, United Kingdom*

[4] *Vakgroep Wiskundige Analyse, Ghent University, Galglaan 2, Blok S22, B-9000 Gent, Belgium*

[5] *Department of Mathematics, Faculty of Science, Ege University, 35100 Bornova-İzmir, Turkey*

e-mails: zpinar@nku.edu.tr, abhishek.dutta@kuleuven.be, a.majumder@lboro.ac.uk, denis.constales@ugent.be, turgut.ozis@ege.edu.tr

Abstract

Key words: Population balance equation, Aggregation, Breakage, homotopy, asymptotic

An area of considerable industrial concern is the phenomenon of particulate aggregation and breakage in chemical and biochemical process systems such as crystallization [1], fluidization [2] and activated sludge flocculation [3]. Population balance equations (PBEs) are widely used to describe the evolution of the particle size distribution (PSD) in the above mentioned processes [1, 3]. These PBEs are integro-partial differential equations which seldom have analytical solutions. This study details several homotopy approaches namely, the Homotopy Perturbation Method (HPM), the Optimal Homotopy Asymptotic Method (OHAM) and the Homotopy Analysis Transform Method (HATM) to obtain approximate analytical solutions for the Population Balance Equation (PBE) involving particulate aggregation and breakage. Using symbolic computation several case studies have been considered and the numerical results have been compared with the analytical solutions obtained from the literature.

The most common approximate solutions for differential equations in the literature involve various asymptotic expansions. There are, however, practical problems associated with this. Instead of trying to develop the asymptotic solutions of a differential equations, it is often more convenient to find an integral expression for the solution and then seek an asymptotic expansion afterwards. This thought gives rise to some novel methods such as Homotopy Perturbation method (HPM) [5] and more recent frequently used variants such as Optimal Homotopy Asymptotic method (OHAM) [6] and Homotopy analysis transform method (HATM) [7], which are based on the same homotopy theory.

We consider a problem with simultaneous aggregation and breakage for which analytical solution is available [4]. As can be seen in Figure 1, solutions provided by all the methods match reasonably well with the analytical solution. In Figure 1, it can be observed that a comparatively better match with the analytical solution is obtained via OHAM. This is because OHAM additionally uses the best approximation (or optimal error) mechanism which is obtained by optimizing the residual to determine fabricated parameters. OHAM may be used for other more complex realistic physical problems in the future.

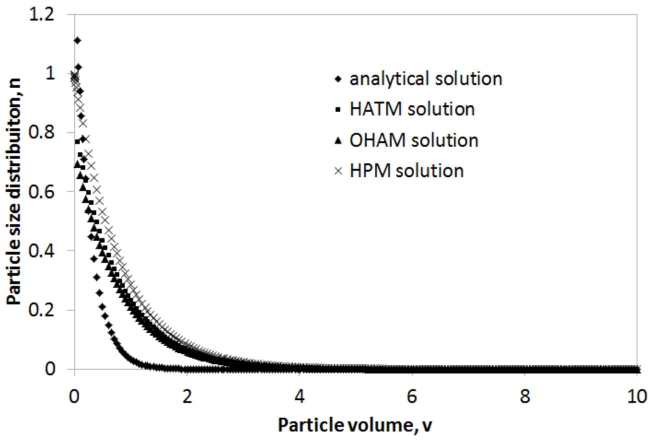

Figure 1: The solution are given for $S = 0.25$ at $t = 1$.

References

[1] A. Majumder, V. Kariwala, S. Ansumali, A. Rajendran, *Lattice Boltzmann method for population balance equations with simultaneous growth, nucleation, aggregation and breakage*, Chem. Eng. Sci. **69(1)** (2012) 316–328.

[2] A. Dutta, D. Constales, R. Van Keer, G. Heynderickx, *Implementation of homotopy perturbation method to solve a population balance model in fluidized bed*, Int. J. Chem. React. Eng. **11(1)** (2013) 1–12.

[3] I. Nopens, T. Koegst, K. Mahieu, P. Vanrolleghem, *PBM and activated sludge flocculation: From experimental data to calibrated model* AIChE J. **51** (2005) 1548–1557.

[4] B. J. McCoy, G. Madras, *Analytical solution for a population balance equation with aggregation and fragmentation*, Chem. Eng. Sci. **58** (2003) 3049–3051.

[5] J.-H. He, *Homotopy perturbation technique*, Comput. Methods Appl. Mech. Engng. **178 (3/4)** (1999) 257–262.

[6] V. Marinca, N. Herinşanu, *Application of Optimal Homotopy Asymptotic Method for solving nonlinear equations arising in heat transfer*, Int. Commun. Heat and Mass **35** (2008) 710–715.

[7] S. Kumar, J. Singh, D. Kumar, S. Kapoor, *New homotopy analysis transform algorithm to solve volterra integral equation*, Ain Shams Engineering Journal (2013), In press.